Kepler's Third Law says that the cube of the satellite's orbit radius is directly proportional to the square of its orbit period. The proportionality constant, c, depends only on the mass of the planet that the satellite (or moon) is orbiting. For distances measured in meters, periods measured in seconds, and masses measured in kilograms, the proportionality constant for satellites orbiting Earth is just

$$C = 1.7 \times 10^{-12} M$$

Problem 1 – What is the equation described by the paragraph above?

Problem 2 – Solve the equation for M – the mass of Earth.

Problem 3 – For objects near Earth, it is convenient to measure their distances in multiples of Earth's radius so that 1.0 Re = 6,378 kilometers. It is also more convenient to use hours as a measure of orbit period. Re-write your equation so that it gives the mass of Earth in kilograms, in terms of the orbit period in hours, and the distance in multiples of Earth's radius.

Problem 4 – The Van Allen Probes spacecraft will be in orbits with a period of 9 hours, and a distance of 3.4 Re. What would you estimate as the mass of Earth given these spacecraft parameters?

Problem 1 – What is the equation described by the paragraph above?

Answer: After substituting for the constant, C, you get
$$R^3 = 1.7\text{x}10^{-12} \text{ M T}^2$$

Problem 2 – Solve the equation for M – the mass of Earth.

$$M = 5.9\text{x}10^{11} \text{ } R^3/T^2$$

Problem 3 – For objects near Earth, it is convenient to measure their distances in multiples of Earth's radius so that 1.0 Re = 6,378 kilometers. It is also more convenient to use hours as a measure of orbit period. Re-write your equation so that it gives the mass of Earth in kilograms, in terms of the orbit period in hours, and the distance in multiples of Earth's radius.

Answer:
$$M = 5.9\text{x}10^{11} (6378000)^3/(3600)^2 \text{ } R^3/T^2$$
$$M = 1.2 \text{ x } 10^{25} \text{ } R^3/T^2$$

Problem 4 – The Van Allen Probes spacecraft will be in orbits with a period of 9 hours, and a distance of 3.4 Re. What would you estimate as the mass of Earth given these spacecraft parameters?

Answer : $M = 1.2 \text{ x } 10^{25} (3.4)^3/ (9.0)^2$
$$M = 5.8 \text{ x } 10^{24} \text{ kilograms}$$

The actual value is 5.98 x 10^{24} kg.